MARS IONOSPHERE SOUNDING BY MARSIS SUBSURFACE SIGNALS ANALYSIS

G. Picardi, A.Masdea, M.Cartacci, M.Iorioi, R. Seu Infocom Dept. - University of Rome "La Sapienza", Italy - E-mail: picar@infocom.ing.uniroma1.it **J.J.Plaut, W.T.K. Johnson, R.L.Jordan, A.Safaenili**
Jet Propulsion Laboratory - Pasadena, CA – USA – **A.Gurnett** Dept. of Physics and Astronomy - University of Iowa - Iowa City, USA - **D.Colonnese, O.Bombaci, E.Zampolini** Alenia Spazio - Space Division - Rome – Italy - E-mail: e.zampolini@roma.alespazio.it

Abstract
According to the Mars Express mission , the MARSIS primary *scientific objectives* are to *map the distribution of water, both liquid and solid, in the upper portions of the crust of Mars*.Moreover three *secondary objectives* are defined for the MARSIS experiment: *subsurface geologic probing, surface characterization,* and *ionosphere sounding.*

According to the last scientific objective, this paper provides a description of the plasma frequency and extra time delay estimation versus Solar Zenith Angle, taking into account of the phase distortion arising from the Ionosphere of the subsurface return signals. As matter of fact this distortion entails a delay, an increase of the sidelobes level, a distortion of the waveform shape and a loss of signal to noise ratio. therefore adaptive compensation of these effects, by Contrast Technique, is implemented in MARSIS in order to optimize the subsurface signal analysis, which entails the estimation of the some Ionosphere Parameters.

1. Introduction

In this paper we report the MARSIS signal analysis that has been performed in order to evaluate the values of the ionosphere plasma frequency as a function of the solar zenith angle. While are available the ionosphere soundings obtained from a low-frequency transmitted waveform this analysis is obtained starting from the ionosphere models and by the evaluation of the significant parameters of the processed echoes in the bands utilized for the subsurface sounding. By considering that:

- The propagation effects of MARSIS signal, through a dispersive medium , such as Mars Ionosphere, was investigated.
- The phase distortion due to the Ionosphere produces (after matched filter, designed not taking into account Ionosphere effects) a delay, an increase of the side lobes level, a distortion of the waveform shape and as a consequence a loss of signal to noise ratio.
- The adaptive compensation (by Contrast Technique) of the previous mentioned effects is required for the subsurface signal analysis, and entails the estimation of the some Ionosphere Parameters.

2. MARS ionosphere

The Martian ionosphere is expected to have the following characteristics:

- The Martian ionosphere is expected to have a strong influence on the performance of the sounder.
- The ionosphere appears as a perfect reflector of RF signal for the frequencies below the plasma frequency.
- A typical profile of the electron plasma frequency in the Martian ionosphere is shown in figure 1.
- The maximum plasma frequency varies with the solar zenith angles as shown in figure 2.
- The best time to operate MARSIS, as a sounder, is at night (Solar zenith angles greater than 90 Degrees).
- Operating above the maximum plasma frequency allows the RF energy to reach the surface. However in the proximity of the plasma frequency significant dispersion as well attenuation are introduced. This dispersion is reduced as the frequency is increased above the plasma frequency. A compensation for this attenuation and dispersion must be performed.

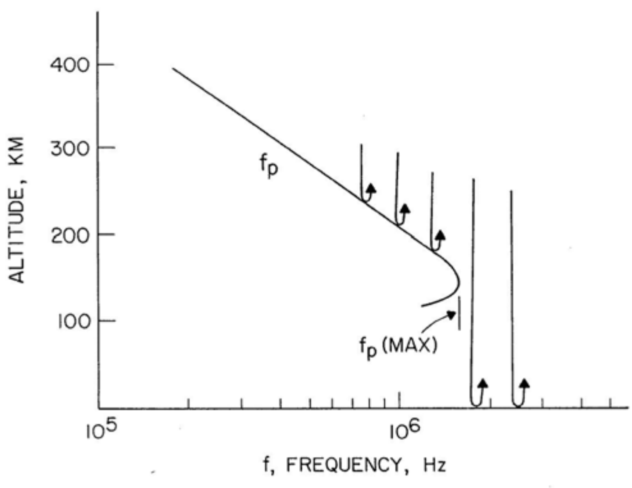


Figure 1

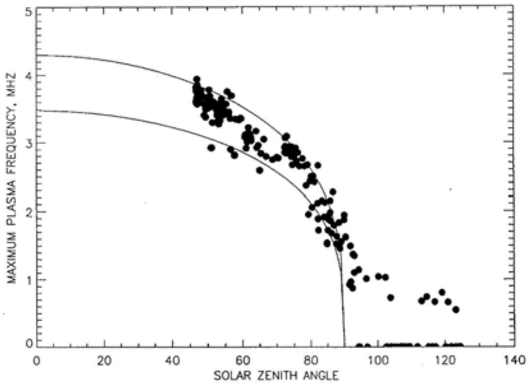


Figure 2

The characteristics of the ionosphere are related to the thickness "l" (L=2l is the two way thickness of the Ionosphere) and to the plasma frequency "f_p "

The plasma frequency is related to the local electron density n_e by the following

(1) $f_p = 8980\sqrt{n_e}$

(n_e is expressed in electron/cm^3 and f is expressed in Hz).

The refraction index n(f) of a ionized plasma, with plasma frequency f_p, is a function of the plasma and propagating frequency as follows:

$$n(f) = \sqrt{1 - \left(\frac{f_p}{f}\right)^2}\, t$$

(2)

If a radio-frequency pulse propagates through a section of such a medium of thickness L it will be subject to an extra-phase shift with respect to the free-space phase shift, which can be expressed by the following:

(3)

$$\Delta\phi(f) = \frac{4\pi}{c} f \int_0^L \left[\sqrt{1 - \left(\frac{f_P(s)}{f}\right)^2} - 1\right] ds$$

being $f_p(s)$ the vertical profile of the plasma frequency and c the speed of light in vacuum. Moreover

(4) $f_0 - B/2 < f < f_0 + B/2$

where f_0 is the central frequency and B is the bandwidth. The following considerations have to be taken into account:

- Those frequencies which are lower than f_p would give an imaginary refraction index, meaning total reflection of the signal.
- For frequencies which are higher than f_p the ionosphere will act as a dispersive medium producing a phase shift which is a non-linear function of the frequency of the propagating signal.

In order to quantify the amount of distortion due to the ionosphere propagation the "gamma" model was introduced to characterize the distribution of the plasma frequency vs. height:

(5)

$$f_P(s) = f_{P,MAX} \frac{s - h_0}{b} e^{1 - \frac{s - h_0}{b}} \delta_{-1}(s - h_0)$$

Moreover an alternative simplified expression for the same phase dispersion, based on an equivalent ionosphere model characterized by a constant plasma frequency $f_{p,EQ}$ and thickness L_{EQ} is given by the following expression:

(6)

$$\Delta\phi_{EQ}(f) = \frac{4\pi L_{EQ}}{c} f \left(\sqrt{1 - \left(\frac{f_{P,EQ}}{f}\right)^2} - 1\right)$$

The typical equivalent parameters can be summarized as follows:

- Night Time Eq. Model (f_0=1.8 MHz) →

$f_{p,EQ}$ < 0.88 MHz *($f_{p,MAX}$ < 1 MHz)*

50 Km < L_{EQ} < 120 Km *(20 Km < b < 50 Km)*

- Day Time Eq. Model (f_0=4.0 MHz) →

$f_{p,EQ}$ < 3.00 MHz *($f_{p,MAX}$ < 3.2 MHz)*

35 Km < L_{EQ} < 80 Km *(20 Km < b < 50 Km)*

The two way phase shift introduced on the signal during the propagation through the ionosphere can be expressed as follows :

(7)

$$\phi_I(f) = \frac{2\pi f L_{EQ}}{c} \sqrt{1 - \left(\frac{f_{p,EQ}}{f}\right)^2} = \frac{2\pi L_{EQ}}{c} \sqrt{f^2 - f_{p,EQ}^2}$$

For a free space layer with the same thickness L, the corresponding phase shift is:

$$\phi_0(f) = \frac{2\pi f L_{EQ}}{c} = 2\pi\tau_0 f$$

(8)

where c is the speed of light and $\tau_0=L_{eq}/c$ is the two way time delay.

Therefore the phase distortion term due to the ionosphere can be written as follows :

(9)

$$\delta\phi_I(f)=\phi_I(f)-\phi_0(f)=2\pi\tau_0\left(\sqrt{f^2-f_p^2}-f\right)$$

In order to have a preliminary evaluation about the effects of the Phase Distortion, numerical solutions have been worked out for various cases of interest. The HANNING Weighting function was applied to properly reduce the time side lobes.

The effects of the phase distortion on the system response are:

1) Time shift
2) Peak loss
3) Widening of the main lobe

3. Compensation of the phase distortion (*)

By considering the Equivalent Uniform Ionosphere Model, the phase distortion term due to Ionosphere can be written as follows:

(10)

$$\delta\phi_I(f)=2\pi\tau_0\left(\sqrt{f^2-f_p^2}-f\right)=\sum_{n=0}^{\infty}a_n(f-f_0)^n$$

We can write by expansion (limited to the third order term) :

(11)

$$\delta\phi_I(f)\cong a_0+a_1(f-f_0)+a_2(f-f_0)^2+a_3(f-f_0)^3$$

where

$$a_0=2\pi\tau_0\left(\sqrt{f_0^2-f_p^2}-f_0\right)$$

[rad]

$$a_1=2\pi\tau_0\left(\frac{f_0}{\sqrt{f_0^2-f_p^2}}-1\right)$$

[rad/MHz]

$$a_2=-2\pi\tau_0\left(\frac{f_p^2}{2(f_0^2-f_p^2)^{\frac{3}{2}}}\right)$$

[rad/MHz^2]

$$a_3=2\pi\tau_0\left(\frac{f_0f_p^2}{2(f_0^2-f_p^2)^{\frac{5}{2}}}\right)$$

[rad/MHz^3]

4. Coarse estimation of the coefficients

To compensate the phase distortion, the coefficients a_i should be known, nevertheless the Ionosphere parameters "l" and "f_p" are not exactly known. For this reason we need to estimate the coefficients a_1, a_2 and a_3.

Linear term "a_1" [rad/mHz]

The extra time delay [sec] corresponding to the frequency f_0 is given by:

(12)

$$\tau=\frac{a_1}{2\pi}=\tau_0\left(\frac{1}{\sqrt{1-\frac{f_p^2}{f_0^2}}}-1\right)=\tau_0\varphi\left(\frac{f_p}{f_0}\right)\approx\tau_0\frac{f_p^2}{2f_0^2}$$

Therefore the total time delay which include the free space layer and Ionosphere effects is:

$$\tau_{TOT}=\tau_0+\tau=\tau_0\left(1+\frac{\tau}{\tau_0}\right)$$

(13)

Moreover we can write

(14)

$$\frac{a_1}{2\pi\tau_0}=\frac{f_0}{\sqrt{f_0^2-f_p^2}}-1=\frac{\tau}{\tau_0}\rightarrow\frac{1}{1-\left(\frac{f_p}{f_0}\right)^2}=\left(1+\frac{\tau}{\tau_0}\right)^2\rightarrow\left(\frac{f_p}{f_0}\right)^2=1-\frac{1}{\left(1+\frac{\tau}{\tau_0}\right)^2}$$

$$\frac{\tau_{TOT}}{\tau_0}=\frac{f_0}{\sqrt{f_0^2-f_p^2}}$$

In order to avoid the influence of surface behavior, to estimate a_1 we can use also the measurement of time delay difference ($\Delta\tau_M$) between the time delay of two signal with different central frequencies f_0 , $f_{02}=f_0+\Delta f$.

(15)

$$\Delta\tau_M=\tau-\tau_2=\tau_0\left[\varphi\left(\frac{f_p}{f_o}\right)-\varphi\left(\frac{f_p}{f_{o2}}\right)\right]=\tau_0\Delta\varphi$$

with $\frac{\tau}{\tau_0}=\varphi\left(\frac{f_p}{f_0}\right)$ and $\frac{\tau_2}{\tau_0}=\varphi\left(\frac{f_p}{f_{02}}\right)$

Assuming an exact estimation of $\Delta\tau_M$ we can write:

$$\tau = \Delta\tau_M \frac{\varphi\left(\frac{f_p}{f_o}\right)}{\Delta\varphi} = \Delta\tau_M \frac{\frac{1}{\sqrt{1-\left(\frac{f_p}{f_0}\right)^2}}-1}{\frac{1}{\sqrt{1-\left(\frac{f_p}{f_0}\right)^2}}-\frac{1}{\sqrt{1-\left(\frac{f_p}{f_{02}}\right)^2}}} = \Delta\tau_M \cdot \gamma = \frac{a_1}{2\pi} = \tau_0\left[\frac{1}{\sqrt{1-\left(\frac{f_p}{f_0}\right)^2}}-1\right]$$

(16)

↓

$$\frac{\Delta\tau_M}{\tau_0} = \frac{1}{\sqrt{1-\left(\frac{f_p}{f_0}\right)^2}} - \frac{1}{\sqrt{1-\left(\frac{f_p}{f_0}\right)^2\left(\frac{f_0}{f_{02}}\right)^2}} \rightarrow f_p = f_0\sqrt{x}$$

Moreover the estimation procedure can be reduced to the estimation of a_2 (a_1 gives only delay time and can be taken into account in range tracking phase), as matter of fact the impulsive response remain approximately unchanged.

Quadratic term "a_2" [rad/MHz^2]

In order to estimate the quadratic term (that is the most important term of the phase dispersion after the linear one) we use the contrast method, which allows also the estimation of f_p. It has to be highlighted that:

1. According to the single parameter equivalent model from the term a_2 we can obtain the equivalent plasma frequency and then the term a_3

2. For low value of the ratio fp/fo we can state that $a_3 \approx -a_2/fo$. This last solution is easier to implement than the preceding one but gives greater error when the plasma frequency is close to the transmitted frequency (especially during the daytime when we operate with the higher frequencies).

To estimate a_2 we use also the measurement of time delay interval. From the theoretical value of the quadratic term we have:

(17)

$$a_2 = -2\pi\tau_0\left(\frac{f_p^2}{2\sqrt{(f_0^2-f_p^2)}\ (f_0^2-f_p^2)}\right) = -\frac{2\pi\tau_0}{f_0}\left(\frac{f_p^2}{2f_0^2}\right)\left(\frac{\tau}{\tau_0}+1\right)^{\frac{3}{2}} = -\frac{2\pi\tau_0}{2f_0}\left(\left(1+\frac{\tau}{\tau_0}\right)^{\frac{3}{2}} - \frac{1}{\sqrt{\left(1+\frac{\tau}{\tau_0}\right)}}\right) =$$

$$= -\frac{\pi\tau_0}{f_0}\left(\left(\frac{\tau_{TOT}}{\tau_0}\right)^{\frac{3}{2}} - \frac{1}{\sqrt{\left(\frac{\tau_{TOT}}{\tau_0}\right)}}\right) = -\frac{\pi\tau_0}{f_0}\left(\frac{\left(\frac{\tau_{TOT}}{\tau_0}\right)^2 - 1}{\sqrt{\left(\frac{\tau_{TOT}}{\tau_0}\right)}}\right) = -\frac{\pi}{f_0}\left(\frac{\tau_{TOT}{}^2 - \tau_0^2}{\sqrt{\tau_0\tau_{TOT}}}\right)$$

We can write also with reasonable approximation:

$$a_2 = -\frac{\pi L}{c}\frac{f_p^2}{f_0^3}\frac{1}{\left(1-\frac{f_p^2}{f_0^2}\right)^{\frac{3}{2}}} = -\pi\tau_0\frac{f_p^2}{f_0^3}\frac{1}{\left(1-\frac{f_p^2}{f_0^2}\right)^{\frac{3}{2}}} \cong -\pi\tau_0\frac{f_p^2}{f_0^3} = -\frac{2\pi\tau_0\Delta\varphi}{f_0}\gamma = \hat{a}_2$$

$$\left(1-\frac{f_p^2}{f_0^2}\right)^{-\frac{3}{2}} \cong 1 \tag{18}$$

where the coefficient a_2 has been expanded around $f_p/f_0=0$

In conclusion it is possible to observe that the value of the quadratic coefficient can be estimated by the linear coefficient as follows

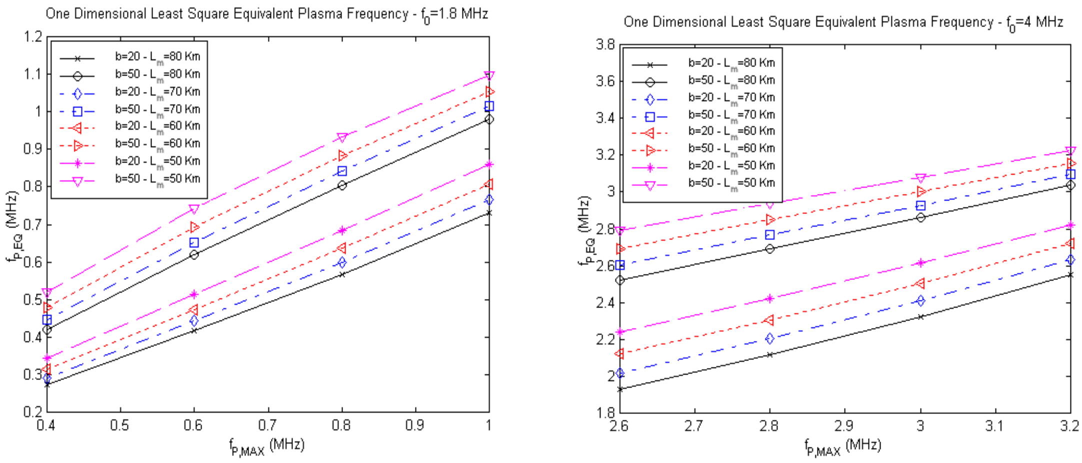
$$\hat{a}_2 = -\frac{\hat{a}_1}{f_0} \tag{19}$$

By the following equation we can estimate the plasma frequency:

$$a_2 = -2\pi\tau_0\left[\frac{f_p^2}{2\left(f_0^2 - f_p^2\right)^{\frac{3}{2}}}\right] \rightarrow \quad \frac{a_2 f_0}{\pi\tau_0} = -\left(\frac{x}{(1-x)^{\frac{3}{2}}}\right) \rightarrow f_p = f_0\cdot\sqrt{x} \tag{20}$$

We can notice that the contrast method is compensating the ionosphere induced dispersion utilizing the single parameter equivalent model by estimating the minimum square error equivalent parameters from the received data and then using the estimated parameters into the equivalent uniform model with negligible approximation error. In order to further reduce the complexity of the compensation procedure it was possible to consider the one-dimensional version of the uniform equivalent model where the ionosphere equivalent thickness is fixed to an average value $\mathbf{L_{EQ}=L_m}$ and the least square optimization is obtained by the single $\mathbf{f_{p,EQ}}$. This further simplification will introduce, of course, additional approximation error with respect to the two-dimensional solution. It has to be considered that it can be the only viable solution when limited processing resources are available. The selection of L_m must be made in a way that is minimizing the residual phase dispersion over the assumed range of variation of the present ionosphere parameters.

By the previous considerations the value of L_{eq}, for our assumed ionosphere parameters, has a range of values between 35 Km and 120 Km. Hence the average value in this case would be about L_m=80 Km ($\boldsymbol{\tau_0}$ = 533 μsec). Taking into account the best, well known, ionosphere gamma model (defined trough the parameters b and $f_{P,MAX}$) the figure 3 shows the 1-D Least Square estimates of $f_{p,EQ}$ with fixed $L_{EQ}=L_m$.



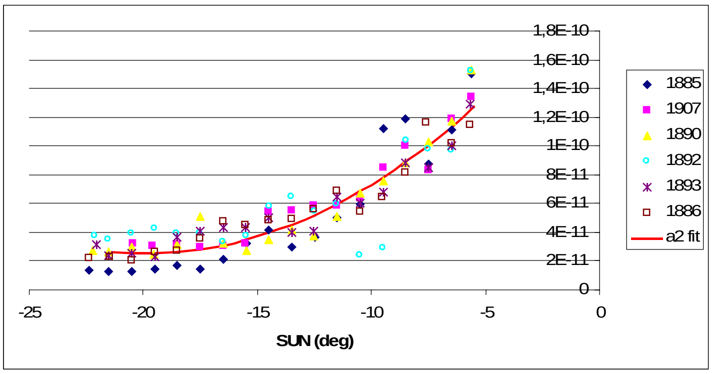
Nightside **Dayside**

Figure 3

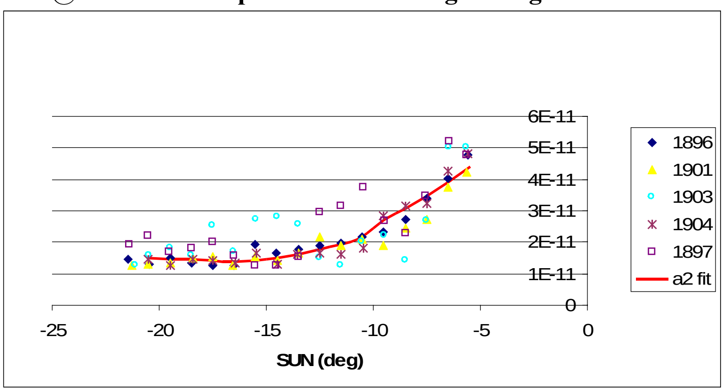
5. Plasma Frequency estimation

Taking into account the available Surface and Subsurface return echoes in night side time of the orbits in July 2005 (MTP 15), in the attached figures are shown the a_2 and $\Delta\tau$ (the difference between the radar time delay and the theoretical delay due to only altitude of S/C information, in the hypothesis of related Mars altitude equal zero) vs. Sun Elevation (SE) Angle. In the following many evaluations of plasma frequency are performed, taking into account the operative mode in each orbit.

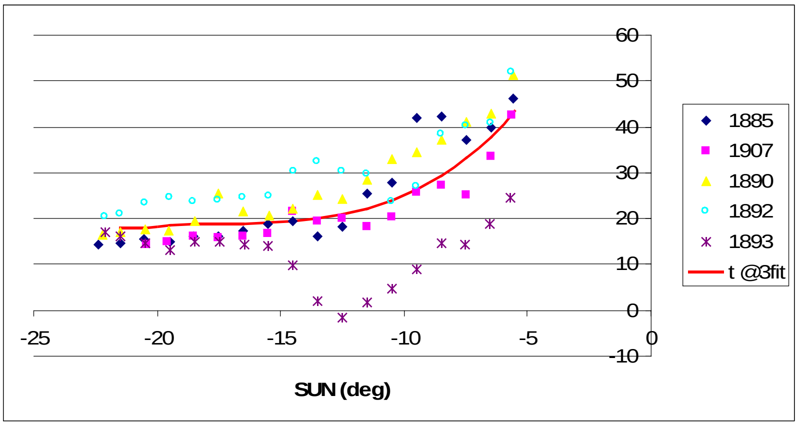
a2 @ 3 MHz : comparison with fitting average.



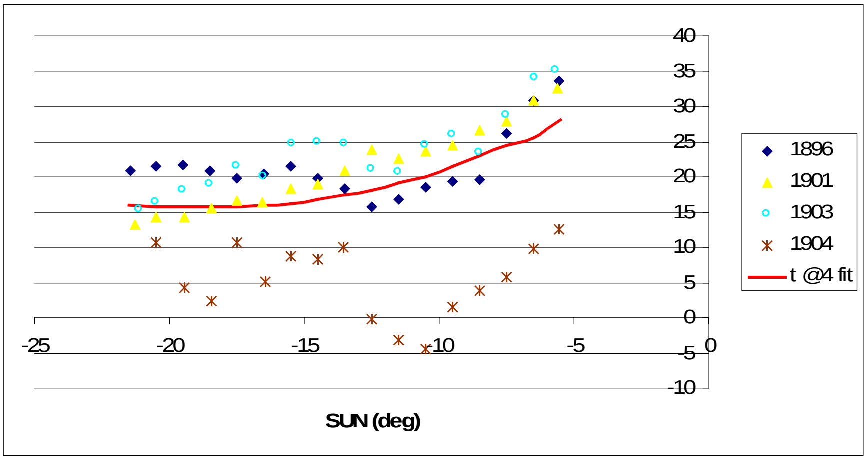
a2 @ 4 MHz : comparison with fitting average.



Δτ @ 3 MHz : comparison with fitting average.



Δτ @ 4 MHz : comparison with fitting average.



Therefore by the best fitting of the a2 and Δτ parameters, we have performed plasma frequency estimation by

$$\frac{a_2 f_0}{\pi \tau_0} = -\left(\frac{x}{(1-x)^{\frac{3}{2}}} \right) \rightarrow f_p = f_0 \cdot \sqrt{x}$$

3 MHz					
SE	**$a_2 \cdot 10^{10}$**	$a_2 f_0 \cdot 10^4$	$\frac{a_2 f_0}{\pi \tau_0}$	x	f_p
- 6.5	**1.15**	**3.45**	**0,2**	**0.1 6**	**1.2**
- 7.5	**0.98**	**2.94**	**0,17**	**0.1 4**	**1.12**
-10.5	**0.64**	**1.92**	**0.11**	**0.1**	**0.94**
-12. 5	**0.5**	**1.5**	**0.09**	**0.0 8**	**0.84**
-15.5	**0,38**	**1.14**	**0.07**	**0.0 6**	**0.75**
-20.5	**0,23**	**0.69**	**0.043**	**0.0 4**	**0.6**

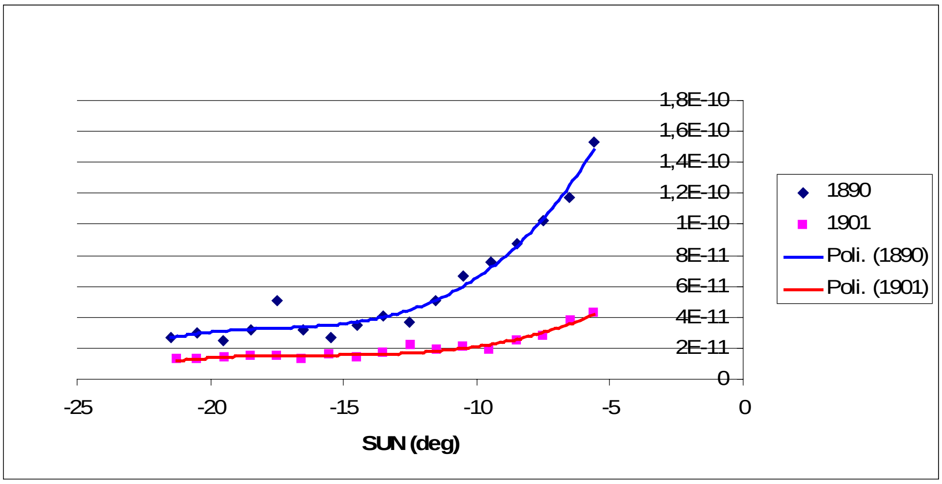
and by

$$\tau = \Delta\tau_M \frac{\varphi\left(\frac{f_p}{f_o}\right)}{\Delta\varphi} = \Delta\tau_M \frac{1-\sqrt{1-x}}{1-\sqrt{\dfrac{1-x}{1-x\left(\dfrac{f_0}{f_{02}}\right)^2}}}$$

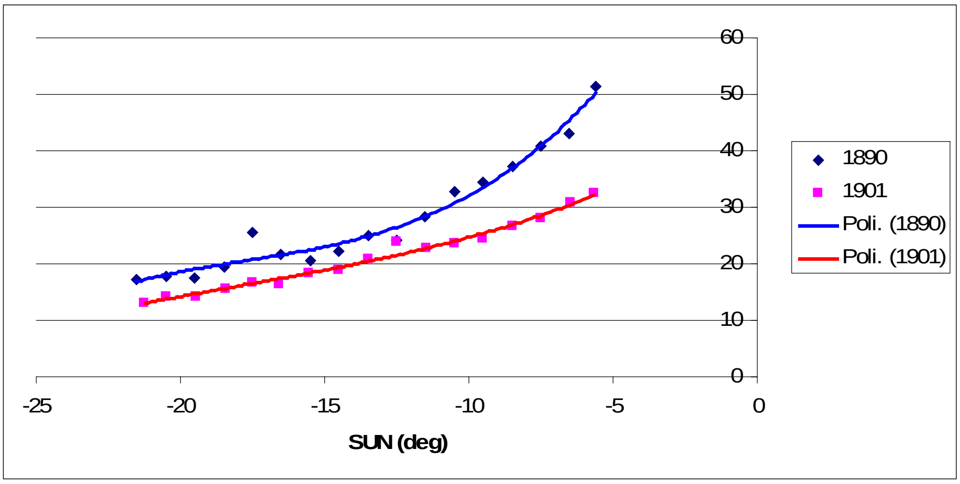
3 4 MHz					
SE	$\Delta\tau$ $\mu\sec\rvert_{3MHz}$	$\Delta\tau$ $\mu\sec\rvert_{4MHz}$	$\Delta\tau_M$ $\mu\sec$	x	f_p
- 6.5	76	50	26	0.17 6	1.26
- 7.5	66	43	23	0.16	1.2
-10.5	48	33	15	0.11	1
-12. 5	42	32	10	0.07 8	0.84
-15.5	38	33	5	0.04 1	0.61
-20.5	36	28.5	7.5	0.05 9	0.73

Moreover best estimation, in order to evaluated also the standard deviation, was performed also by considering the orbit located in the same region (stationary hypothesis)

a2 1890 (3 MHz) - 1901(4 MHz)



Δτ 1890 (3 MHz) - 1901(4 MHz)



SE	$a_2 \cdot 10^{10}$	$\frac{a_2 f_0}{\pi \tau_0}$	χ	f_p	$\Delta\tau$ μsec	Lat
1890 (3 MHz)						
-6.5	1,17	0.21	0.16	1.2	48	26,7
-7.5	1.02	0.18	0.14	1.12	40	29,0
-10.5	0.67	0.12	0.1	0.95	35	36,0
-12.5	0.37	0.066	0.06	0.73	27	40,9
-15.5	0.27	0.048	0.04	0.6	22	48,6
-20.5	0.3	0.053	0.05	0.67	18	63,7
1901 (4 MHz)						
-6.5	0.37	0.088	0.078	1.12	31	30.47
-7.5	0.27	0.064	0.058	0.96	28	32.7
-10.5	0.2	0.047	0.044	0.84	24	39.5
-12.5	0.22	0.052	0.048	0.87	22	44.2
-15.5	0.15	0.035	0.033	0.72	18	51.7
-20.5	0.13	0.031	0.03	0.7	15	65.7

$\overline{f_p}$	$\Delta\tau_M$ μsec	$\frac{\Delta\tau_M}{\tau_0}$	$x = \frac{\Delta\tau_M / \tau_0}{0.22}$	f_p
1.16	**17**	**0.032**	**0.14**	**1.14**
1.04	**12**	**0.022**	**0.10**	**0.96**
0.89	**11**	**0.020**	**0.093**	**0.92**
0.8	**5**	**0.009**	**0.042**	**0.62**

6. Conclusions

In the following tables the mean values of the plasma frequency are shown vs. the Sun Elevation angle. Moreover we can notice the agreement with the well known results available in literature

It is our opinion that the formula used in the last section must be used in the acquisition and tracking phase, at least if the operative mode allows the use of only one frequency in the previous phase

Sun El.	f_p (3 MHz)	f_p (4 MHz)	$\overline{f_p}$ (3-4 MHz)
- 6.5	**1.19**	**1.15**	**1.17**
-7.5	**1.11**	**1.06**	**1.08**
-10.5	**0.94**	**0.87**	**0.91**
-12.5	**0.79**	**0.81**	**0.8**
-15.5	**0.71**	**0.75**	**0.73**
-20.5	**0.62**	**0.74**	**0.68**