Data assimilation for parameter estimation: useful for Mars?

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Parameter estimation

- GCMs typically require specification of O(10^2) parameters
- Parameterizations fall into two kinds of category
  - "Rational" – based on a traceable set of approximations to the exact problem [e.g. radiative transfer for gaseous constituents]
  - "Non-rational" – based on *heuristic* representations with *ad hoc*, empirically adjustable coefficients [e.g. boundary layer mixing, gravity wave drag….]
- All parameters are uncertain to some degree, but….
  - Uncertainties in parameters for "rational" parameterizations can usually be quantified objectively
  - Uncertainties in "non-rational" parameterizations may be difficult to assess from first principles theory – parameters need to be adjusted empirically to optimize agreement between model and observations
- **But How….?**
  - Brute force trial and error…..?
  - But optimal parameters may be state-dependent and depend on time and space?
  - Better to find a more objective approach – adapted from data assimilation

Parameter estimation and DA

- Data assimilation is typically designed for **state estimation**, based on minimization of a cost function:
  - $J(\boldsymbol{x}) = (\boldsymbol{x}-\boldsymbol{x_b})^{\boldsymbol{T}}\boldsymbol{B}^{-\mathbf{1}}\,(\boldsymbol{x}-\boldsymbol{x_b}) + (\boldsymbol{y}-H[\boldsymbol{x}])^{\boldsymbol{T}}\boldsymbol{R}^{-\mathbf{1}}\,(\boldsymbol{y}-H[\boldsymbol{x}])$

  for dynamical system represented by
- $\boldsymbol{x}^{n+1} = f[\boldsymbol{x}^{n};\boldsymbol{\lambda_i}]$; where λ_i are parameters
- But we can also treat parameters as variables in the system
  - $\boldsymbol{x}^{n+1} = f[\boldsymbol{x}^{n};\boldsymbol{\lambda^n}]$
  - $\lambda^{n+1} = \lambda^{n}$

  and minimize the cost function by varying λ as well as **x.**

BUT

- for highly nonlinear systems, *J(**x**)* may have multiple local minima – need sophisticated minimization algorithms?
- Observations may not be well correlated with λ (so *J* not much affected by λ) – value of λ uncertain but perhaps not critical?
- Optimal λ may be state dependent…… - need state-dependent DA?

Methods for DA parameter estimation

- 4D-var – minimizes a J(**x**) over a time interval
  - Takes state dependence into account
  - Uses clever minimization algorithms for high dimensional dynamical system
  - Needs tangent linear and adjoint versions of full model (including gradients w.r.t. parameters)
- EnKF – and its variations (LETKF etc.)
  - Uses ensembles to estimate uncertainties
  - Takes state dependence into account
  - Relatively straightforward to code and inexpensive to run
- Particle filters (PFs)
  - Doesn’t need to assume Gaussian errors
  - Better at coping with strong nonlinearity
  - Needs many “particles” (i.e. model simulations) to converge – expensive!

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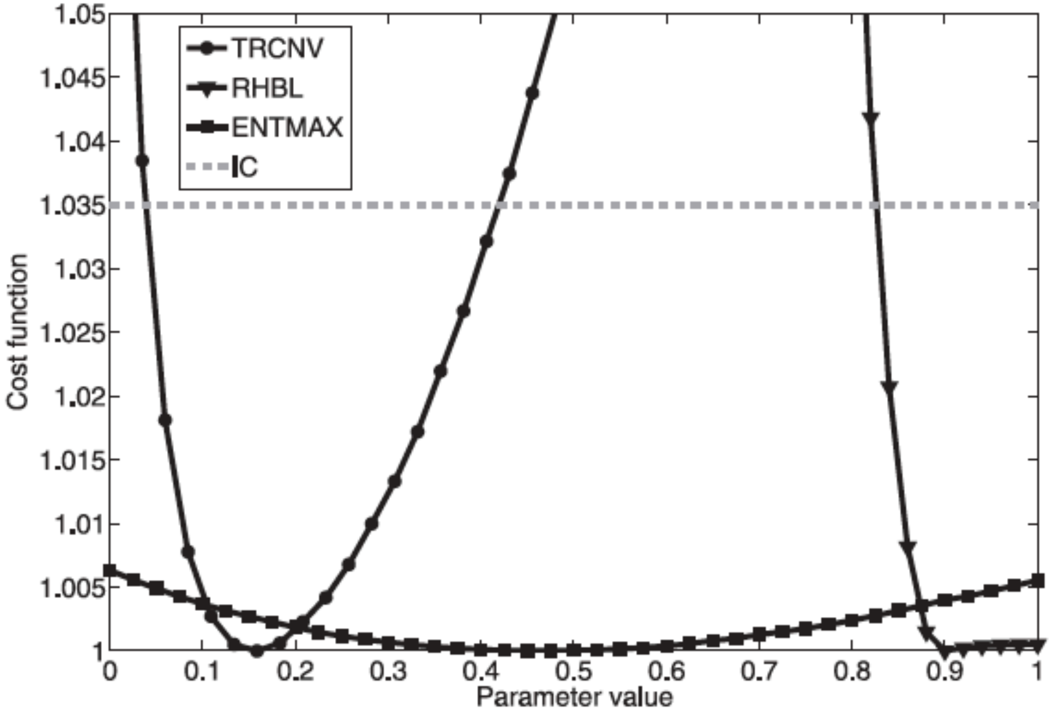
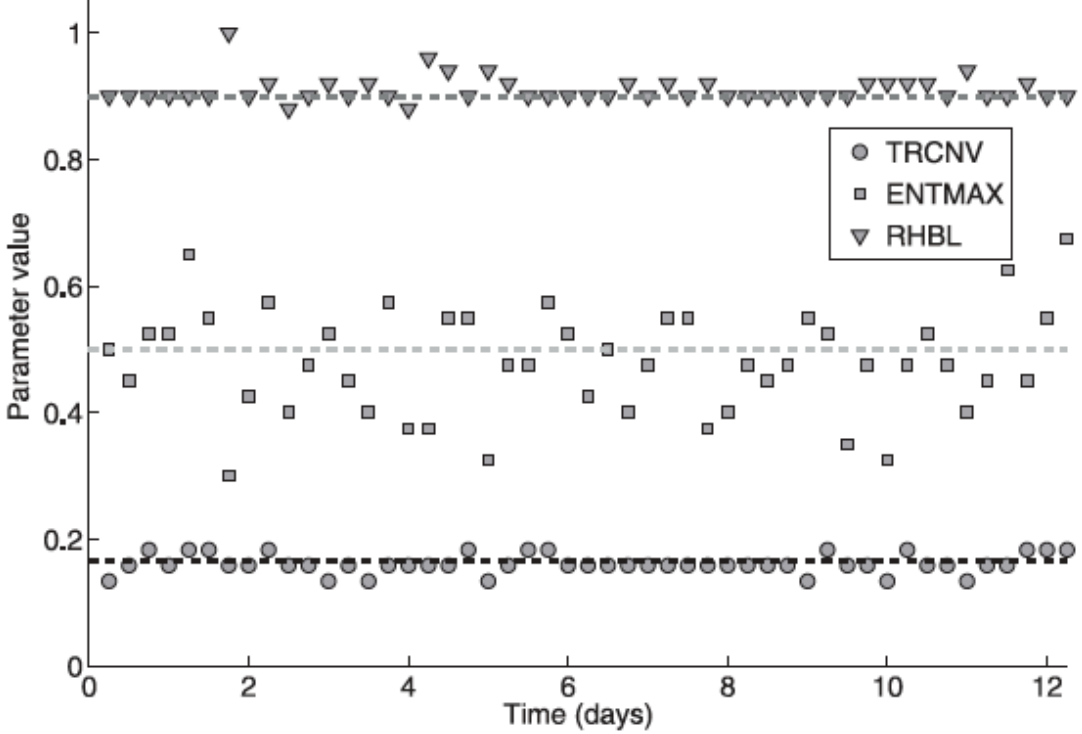
An example [Ruiz et al. JMSJ 2013]

Cost function



Optimal parameter values



- Uses “SPEEDY” intermediate GCM and assimilates for 3 convective parameters
- Based on LETKF assimilation
- Minimises cost function to recover simultaneous optimal values of parameters

Questions?

- Perfect vs imperfect models?
  - Bias correction?
- Interfering effects of other parameters….?
  - Important if set of parameters being optimized is incomplete….
  - Which parameters need to be included….?
- Good for optimizing climate models but may not improve forecasts much….?
- Which parameters would be most important to optimize for Mars models?
- Several groups already using LETKF for assimilation, but….
  - Size of ensembles?
  - How nonlinear?
  - Gaussian assumptions OK…..?