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# Martian Environmental Model

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## WP6 deliverable :

## Technical report: New model radiation schemes, parameterization and validation.

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# New model radiation schemes, parameterization and validation.

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## 1 Introduction

This technical note presents the new model radiation scheme which has been developed at LMD in the frame of extension to ESA Contract 11369/95/NL/JG.

In the previous phase of the contract, the possible extension of the model top up to 120 km has been investigated in some detail. In particular, a way of introducing some non-LTE effects in the infra-red part of the radiative transfer code has been derived and the parameterization for the visible part of the spectrum has been improved.

A good prediction of the atmospheric thermal structure in this altitude range is taking an increasing importance since it appears that aero-breaking or aero-capture will be systematically used for future missions to Mars.

However, a major difficulty has arisen. First results have shown that the radiative cooling rates can reach values of several hundred K per day above 100 km. This make the use of the existing radiative transfer code impracticable. In fact, the radiative transfer code – which has been adapted from the "state-of-the-art" terrestrial code used at the European Center for Medium Range Weather Forecasts – is much too costly to be computed more than about 50 times a day in current configurations. Moreover, the cost of this code increases as the square of the number of vertical layers. Both reasons required a complete rethinking and rewriting of the radiative code.

The presentation of the new radiative code which follows will serve as basis for a paper.

Computation of radiative transfer is a key element in the modeling of atmospheric circulation. Absorption and emission of visible and thermal infrared radiation are the original forcing of atmospheric circulation. With typical horizontal grids of a few thousands to 10.000 points, and since we want to cover years with explicit representation of diurnal cycle, operational radiative codes must be extremely fast. Representation of radiative transfer must be drastically simplified and parameterized.

For Mars, the main contributors to atmospheric radiation are by far carbon dioxide, which represents about 95% of the atmospheric mass and airborne dust particles. Even outside large planetary scale dust-storms, extinction of solar light by dust is of several tenth of percent. Carbon dioxide is dominant at infrared frequencies with a vibration-rotation line spectrum that must be accounted for with extensive details in order to insure accurate enough descriptions of corresponding energy redistribution processes.

In the development phase of the LMD Marian atmospheric circulation model, a major step was the derivation of a radiative transfer code for the $CO_2$ 15 $\mu$m band(Hourdin, 1992). This model was based upon the Wide-band Model approach developed by (Morcrette et al., 1986; Morcrette, 1989), and used in the operational model of the European Center for Medium Range Weather Forecast. This model is based on a two stream flux formulation. Wide band transmitvities are fitted as Padé approximant and as a function of an integrated absorber amount including a simple representation of the temperature dependency of absorption coefficients. For application to Marian atmosphere, the fit was somewhat adapted in order to account for Doppler line broadening which becomes significant above 50 km.

In standard configurations of the Marian model, with a 25 layer vertical discretisation, the infrared computation alone represents a significant part, often up to one half, of the total cost of the General Circulation Model. Similar reports are made concerning terrestrial GCMs. Because of the band nature of the code of the long wave model, transmission functions are not multiplicative anymore and the determination of radiative fluxes at each level requires independent calculations of the contributions of emission of all atmospheric layers. The corresponding cost therefor increases as the square of the number of vertical layers. This quadratic dependency undoubtedly represents a severe limitation when thinking of further GCM refinements, in particular as far as near surface and high atmosphere processes are concerned, both requiring a significant increase of the vertical discretisation.

It is commonly recognized that despite of this formal difficulty, IR radiative transfers are dominated by a few terms such as cooling to space and short distance exchanges. Thus, in practice, the quadratic dependency of standard schemes is widely over-costly, a significant part of the computations resulting in fully negligible contributions. In standard flux formulations however, it is difficult to quantify the relative importance of the various contributions which are not generally identified as such in the code, and difficult also to take advantage of this for code optimization.

Green (1967) suggested that a reformulation of radiative transfer in terms of net exchange, could be help-full to quantify the relative importance of the various terms entering into the radiative budget and help design more efficient models. In Greens approach, the quantity under consideration is directly the net energy exchanged between two atmospheric layers (or more generally two volume of gas or surfaces). Joseph et al. (1976) attempted to use the net exchange approach for modeling radiative transfer in the earth atmosphere. Despite some numerical difficulties, they showed that radiative exchanges between an atmospheric layer and boundaries (space and ground) are dominant, and that exchanges between an atmospheric layer and the rest of the atmosphere are not negligible and contributed to around 15%. Using this formulation and considering some particular cases, Bresser et al. (1995) did some elaborate analytical development in order to build a simplified model adapted to the simplified analytical atmospheric model (study of gravity waves...). Cherkaoui et al. (1996, 1998) used the formulation to build a very efficient Monte-Carlo method, and Dufresne et al. (1998) to analyze in which range of absorption coefficients, $k$, net radiative exchanges are dominant.

Here, we show how this net exchange approach can help deriving much more efficient operational radiative codes for circulation models. Since our goal is firstly to present a new approach, we will not discuss the way we compute transmissions. We will keep exactly the same formulation as in the original model (Hourdin, 1992). Parameterization of Non-LTE effects based on a modified Planck function is developed independently and is discussed in another companion technical report to be delivered for same WP6.

We start with a presentation of the net exchange formulation for radiative transfer and with the associated derivation for an operational code. We illustrate the approach for typical Marian conditions by distinguishing between exchange with boundaries (surface and space), with nearby layers and distant layers. We show in particular that closest layers need a special treatment and suggest an objective criterion for the degree of accuracy required. We then present 3D comparison between old and new code. We show finally that numerical cost can be significantly reduced by computing much less frequently exchanges with distant layers. The cost is then proportional to the number of layers ($N$ instead of $N^2$). Principle are first discussed on 1D calculations performed for typical Martian conditions.

## 2 Net exchange formulation

### 2.1 General approach

The net exchange formulation is firstly an integral formulation in which the radiative budgets are addressed directly without explicit formulation of the radiative intensity field. If the full system is discretized in $N$ surface or volume elements $E_i$, the radiative budget of element $i$ can be formally written as

$$\psi_i = \sum_{i \neq j} \phi_{j,i} - \mathcal{E}_i \tag{1}$$

where $\mathcal{E}_i$ is the total radiative power emitted by $i$ and $\phi_{j,i}$ is the radiative power emitted by $j$ and absorbed by $i$ (exchange). In the net exchange formulation, the emitted power $\mathcal{E}_i$ is no longer considered as a whole but also split into the $N-1$ exchanges toward the other elements as

$$\mathcal{E}_i = \sum_{i \neq j} \phi_{i,j} \ . \tag{2}$$

The radiative budget of element $i$ then writes

$$\psi_i = \sum_{i \neq j} \psi_{i,j} \tag{3}$$

where $\psi_{i,j} = \phi_{j,i} - \phi_{i,j}$ is the individual net exchange of radiative power between $i$ and $j$.

As flux formulations, net exchange formulations are intrinsically energy conserving. In the first case, conservation is obtained from the final differentiation of radiative fluxes. In the net exchange formulation conservation $(\sum_i \psi_i = 0)$ is more constraining relying on the reciprocity of individual exchanges $(\psi_{i,j} = -\psi_{j,i})$. A direct consequence is that different approximations can be used for the various exchanges without violating neither conservation nor reciprocity. More generally, the net exchange formulation is a powerful analysis frame allowing in particular to quantify the relative importance of the various terms. Note finally that, in quasi isothermal conditions, flux formulations ends with the differentiations of a quasi-constant flux whereas net exchange formulations compute the sum of small residuals avoiding some numerical problems.

In the general case, $\psi_{i,j}$ is the sum of partial net exchanges at all frequencies and along all possible optical paths between $E_i$ and $E_j$. Corresponding formulations include complex spectral and optico-geometric integrals that may resumed as (Dufresne et al., 1998)

$$\psi_{i,j} = \int_0^{+\infty} d\nu \int_{\Gamma_{i,j}} d\gamma \tau_\gamma^\nu K_{1,\gamma}^\nu K_{2,\gamma}^\nu \left( B_{1,\gamma}^\nu - B_{2,\gamma}^\nu \right) \tag{4}$$

In this expression, $\Gamma_{i,j}$ is the space of all optical paths with first and second extremities within $E_i$ and $E_j$ respectively. For each optical path, $\tau_\gamma^\nu$ is the total transmittance function along the path, $B_{1,\gamma}^\nu$ is the spectral black body intensity function at the first extremity and $K_{1,\gamma}^\nu$ is either the absorption coefficient (if

$E_i$ is a volume element) or the directional emissivity (if $E_i$ is a surface element) at the first extremity. The index 2 is used similarly for the second extremity. As will be illustrated later, net exchange rates are valuably interpreted along this line: local temperature differences and emission properties define exchange potentials that must be associated with optico-geometric constraints (possible optical paths, extinction) to analyze the physics of infrared radiation exchanges between considered elements.

Net exchange formulations automatically insure two important (and equivalent?) properties of radiative transfer : reciprocity ($\psi_{j,i} = -\psi_{i,j}$) and energy conservation ($\sum_i \sum_j \psi_{i,j} = 0$).

### 2.2 Application to the Martian atmosphere

For Marian GCM studies restricted to the $15\mu m$ $CO_2$band, assumptions can be retained that allow very significant simplifications of the geometric: the atmosphere can be assumed perfectly stratified along the horizontal, "effect mirage" as well as molecular and particulate scattering can be neglected, the boundaries (space and planetary surface) are assimilated to black body surfaces (with a temperature of 0 K for space). Under these assumptions, the space of relevant optical paths reduces to straight lines between exchange positions. Dividing the atmosphere into $N$ layers, net exchange rates can be derived as

- exchange between two layers

$$\psi^{\nu}_{i,j} = \int_{z_{i-\frac{1}{2}}}^{z_{i+\frac{1}{2}}} \int_{z_{j-\frac{1}{2}}}^{z_{j+\frac{1}{2}}} (B^{\nu}_{z_j} - B^{\nu}_{z_i}) \left| \frac{\partial^2 \tau_\nu(z_i, z_j)}{\partial z_i \partial z_j} \right| dz_i dz_j \tag{5}$$

- exchange between a layer and boundaries

$$\psi^{\nu}_{i,b} = \int_{z_{i-\frac{1}{2}}}^{z_{i+\frac{1}{2}}} (B(T_b) - B^{\nu}_{z_i}) \left| \frac{\partial \tau_\nu(z_i, z_b)}{\partial z_i} y \right| dz_i \tag{6}$$

with $T_b = T_{surf}$ for exchange with the planetary surface and $T_b = 0$ K for the cooling to space.

where $z_{i-\frac{1}{2}}$ and $z_{i+\frac{1}{2}}$ are the altitudes at the boundaries of atmospheric layer $i$ and the monochromatic integrated transmission function $\tau_\nu$ is defined as

$$\tau_\nu(z, z') = 2 \int_0^{\frac{\pi}{2}} \exp\left( - \int_z^{z'} k_\nu(z)\rho(z) \frac{1}{\cos\theta} dz \right) \sin\theta \ \cos\theta \ d\theta \tag{7}$$

where $\rho$ is the air density and $k_\nu$ is the monochromatic absorption coefficient.

Computation of such integrals requires three types of numerical integrations.

- The angular integration is computed by applying the diffusive approximation which consists in the use of a mean angle ($1/\cos\theta = 1.66$, see Elsasser, 1942).

| | $\sigma$ | Approximate Height | | $\sigma$ | Approximate Height |
|---|---|---|---|---|---|
| 1 | 0.999907 | 3.6 m | 14 | 0.659718 | 16.1976 km |
| 2 | 0.999579 | 16.4 m | 15 | 0.580298 | 21.1927 km |
| 3 | 0.998980 | 39.8 m | 16 | 0.506077 | 26.5220 km |
| 4 | 0.997893 | 82.1 m | 17 | 0.438807 | 32.0763 km |
| 5 | 0.995924 | 159.0 m | 18 | 0.378806 | 37.8021 km |
| 6 | 0.992380 | 297.9 m | 19 | 0.325576 | 43.6990 km |
| 7 | 0.986050 | 547.0 m | 20 | 0.278323 | 49.8056 km |
| 8 | 0.974939 | 988.4 m | 21 | 0.235920 | 56.2423 km |
| 9 | 0.955975 | 1.7533 km | 22 | 0.197541 | 63.1562 km |
| 10 | 0.925138 | 3.0301 km | 23 | 0.161305 | 71.0479 km |
| 11 | 0.878650 | 5.0378 km | 24 | 0.127463 | 80.2175 km |
| 12 | 0.815669 | 7.9343 km | 25 | 0.084210 | 96.3582 km |
| 13 | 0.740321 | 11.7087 km | | | |

Table 1: $\sigma$ levels and approximate heights based on -10ln($\sigma$)

- The frequency integration is approximated by a wide band model of transmission (Hourdin, 1992). In what follows, the model will be used as a narrow rather than wide band model which means that the planck function is constant enough on the interval so that it can be taken off the integral. This point is discussed further later on.
- The vertical integration is what we concentrate on in the rest of the text with various levels of approximation.

This simplification was shown to produce minor errors in martian computations.

### 2.3 Example of net exchange computation

We present a numerical computation of net exchanges for 25 atmospheric layers (typical vertical discretisation of GCMs) with an enhanced resolution near the surface (see table 1) and for a typical temperature profile derived from Viking entry profiles (Seiff, 1982) and refered to as perturbed profile by Hourdin (1992). The atmosphere is assumed to be dust-free and the surface pressure is fixed to 700 Pa. For this computation, the vertical integrals are computed by dividing each of the 25 layers into 20 sub-layers, taking into account subgrid-scale temperature variations (see Fig. 1).

Fig. 2 displays a graphical representation of the radiative exchange in the atmosphere. More precisely, each element of the matrix displays the intensity of the net exchange rate between each pair $i, j$. This net exchange $\chi_{i,j}$ is expressed in Kelvin per Day:

$$\chi_{i,j} = \frac{g}{Cp}\frac{1}{\delta t}\frac{\psi_{i,j}}{\delta p_i} \tag{8}$$

(where $g$ is the gravity, $Cp$ the massic heat capacity, and $\delta t$ the length of the

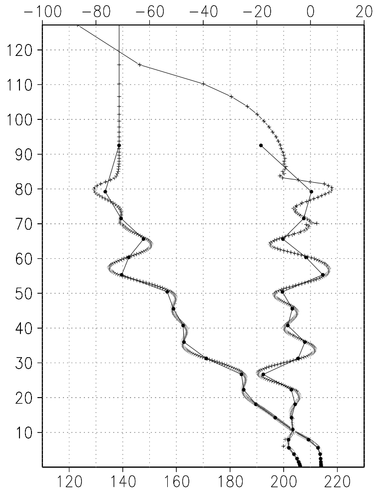


Figure 1: Typical temperature profile. 500 sublayers and 25 layers.

day. The cooling rate of a layer $i$ is:

$$\chi_i = \sum_j \chi_{i,j} \tag{9}$$

As an example for reading Fig. 2, consider layer $i = 10$ (highlighted in the figure). The left curve gives its temperature, the matrix line gives the exchange values $\chi_{i,j}$ between layer $i$ and layers $j$ (including ground and space), and the right curve gives its cooling rate $\chi_i$ which is just the sum of all exchanges values $\chi_{i,j}$ on line $i$ (cf Eq. 9). Layer 10 is heated by the warmer underlying atmosphere and surface and looses energy toward the colder layers above and toward space. Note that the picture is more complex in the upper atmosphere where the wavy structure of temperature tends to be dumped by exchanges between nearby layers (a warmer layers cools toward nearby colder layers).

More generally $\chi_{i,j}$ and $\chi_{j,i}$ are of opposite sign. It means that when layer $i$ loses energy to layer $j$, this layer $j$ gains energy from layer $i$. In fact, the net exchange matrix expressed in $W.m^{-2}$ is exactly anti-symmetrical ($\psi_{i,j} = -\psi_{j,i}$).

More importantly, the net-exchange approach allows to quantify the relative importance of the various terms. The most important loss is the exchange with space: we thus obtain the classical result that the radiative heat budget of a layer is dominated by the cooling-to-space. Further, we see that the net exchange generally decreases with increasing distances. If the two layers under consideration are assumed to be isothermal (allowing to take the Planck functions off the integrals) the net exchange is the product of two terms : 1) difference in Planck functions which generally increases with distance between layers (as temperature difference), and 2), an optical factor $\left|\frac{\partial^2 \tau_\nu(z_i,z_j)}{\partial z_i \partial z_j}\right|$ which is a monotone decreasing function of distance. The figure shows that the net exchange decreases with distance, which means that this decreasing function is the dominating term.

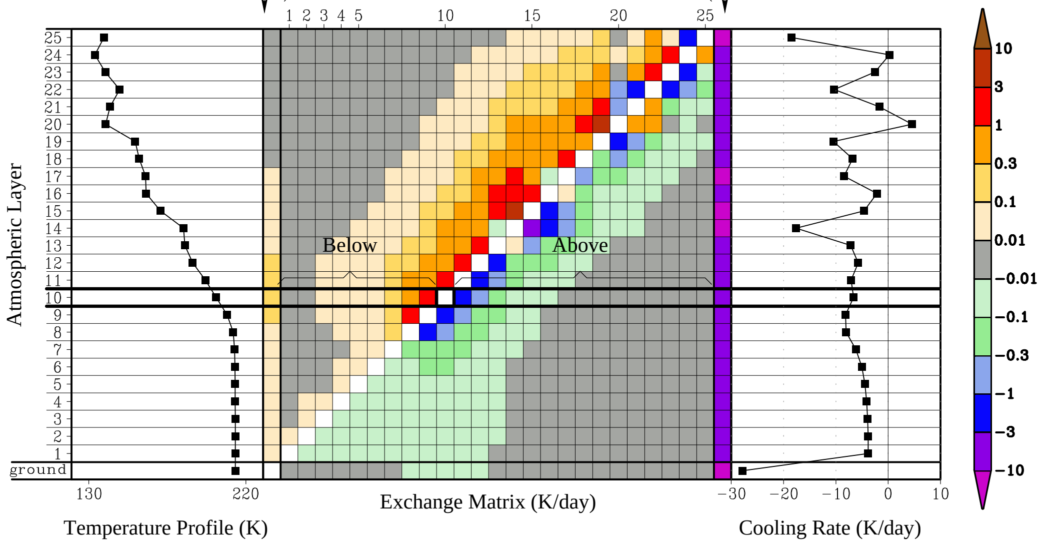


Figure 2: Graphical representation of the radiative exchanges in the Martian atmosphere.

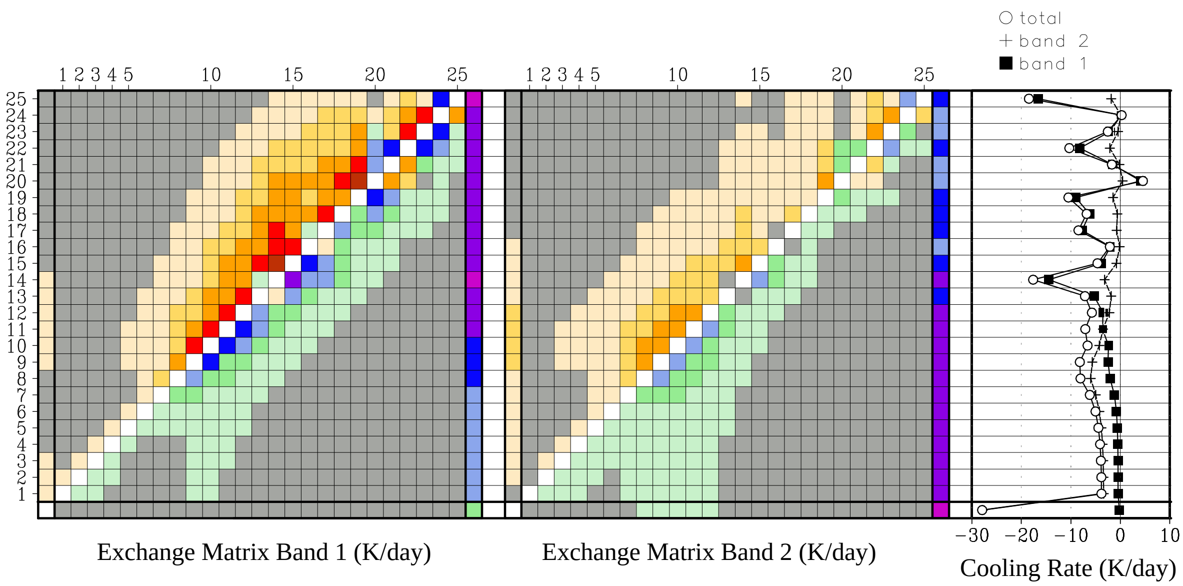


Figure 3: Exchange matrix for each $CO_2$ band and corresponding vertical cooling profiles: total, band 1 and band 2

# 3 Vertical integration for circulation models

The computation of the exchange matrix presented above uses a sub-grid scale description of the temperature profile. In practice, in circulation models, one is faced with the limitation that only the mean temperature of the atmospheric layer is known (here we assume that the temperature variable of the model is representative of the mean temperature rather than of the temperature at the middle of a layer).

## 3.1 Isothermal layers

The first approximation to the vertical integration consists in assuming that this temperature is constant inside each sub-layer in which case exchange between two atmospheric layers (Eq. 5) reduces to

$$\psi_{i,j} = \xi_{i,j} \times (B_i - B_j) \tag{10}$$

where $\xi_{i,j}$ is an optical factor consisting in the difference of transmissions computed between boundaries of the two layers

$$\xi_{i,j} = \tau_{i+\frac{1}{2},j-\frac{1}{2}} - \tau_{i-\frac{1}{2},j-\frac{1}{2}} - \tau_{i+\frac{1}{2},j+\frac{1}{2}} + \tau_{i-\frac{1}{2},j+\frac{1}{2}} \tag{11}$$

At this stage, we can draw a close parallel between net exchange and flux formulations. If we denote by $F_{i\to j+\frac{1}{2}}$ the contribution of layer $i$ to the flux at interface $j+\frac{1}{2}$

$$F_{i\to j+\frac{1}{2}} = B_i \left(\tau_{i+\frac{1}{2},j+\frac{1}{2}} - \tau_{i-\frac{1}{2},j+\frac{1}{2}}\right) \tag{12}$$

$\psi_{i,j}$ is just obtained as the difference of four of those fluxes $F_{i\to j+\frac{1}{2}} - F_{i\to j-\frac{1}{2}} - F_{j\to i+\frac{1}{2}} + F_{j\to i-\frac{1}{2}}$. In the flux formulation, $F_{i\to j+\frac{1}{2}}$ are first summed at a given interface $j+\frac{1}{2}$ and afterward differentiated vertically. Numerical operations are hence exactly the same except in their ordering. However, the advantages of using the net exchange formulation are numerous. In the flux form, the flux are first integrated and then differentiated. When temperature contrasts are weak (here near the surface for instance), the net exchange can be by order of magnitudes smaller than fluxes leading to a loss of accuracy on the difference which may actually lead to violate reciprocity of radiative transfer (the colder layer heating the warmer for instance).

Moreover, although both formulation are $N^2$ on the vertical (in fact it is generally the computation of the $N^2$ transmissions which dominates the cost), the net exchange formulation allows to make some simplification without any chance of violating reciprocity of atmospheric heat transfer. For instance, since the diagonal is dominant (as shown above) one can hope to save a large amount of CPU time by approximating the non diagonal terms (for instance by computing them less frequently as shown below).

Note that, because of this exact equivalence, the net exchange formulation can also been used to diagnose energy exchanges in a flux formulation.

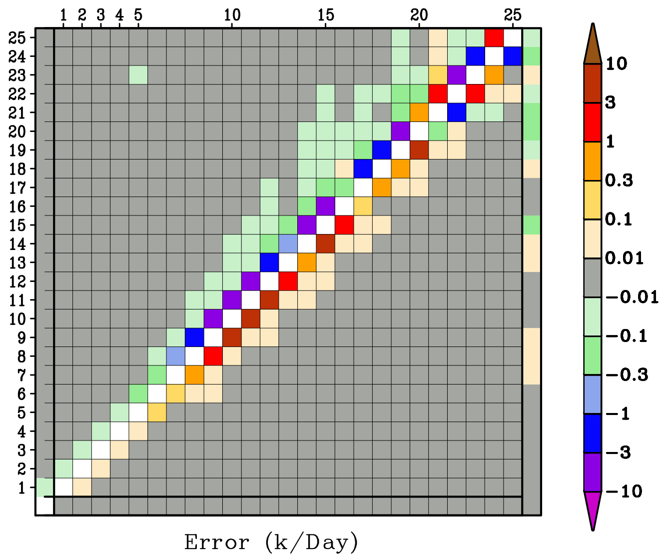


Figure 4: Error matrix with the formulation based on isothermal layers. The error is computed with respect to the reference computation performed with 20 sub-layers inside each layer. Line $i$ gives the error in K/day for the cooling rate of layer $i$ due to its exchange with layer $j$.

## 3.2 Special treatment of exchanges with nearby layers

The isothermal formulation described above leads to significant errors, especially for exchanges between nearby layers as illustrated in Fig. 4. The errors obtained close to the diagonal are generally even greater than the exchange itself. The reason is rather simple. Because the Martian atmosphere is extremely opaque in the $CO_2$ band, the isothermal computation leads to exchanges between layer $i$ and $i+1$ of the order of $B_i - B_{i+1}$. In fact, the true exchange is the difference of the Planck function computed at a distance from the interface corresponding to an optical depth unity.

Therefore, a special treatment was applied to the exchange with nearby layers. This treatment consists in splitting each layer in sub-layers. The temperature of each sub-layer is computed assuming a linear profile for the two exchanging layers.

Fig. 5 presents errors in Kelvin per Day for exchanges between layer $i$ and $i+1$ computed with different numbers of sub-layers.

We finally retained a discratisation with 3 sub layers. Fig. 6 shows the error matrix with this special treatment.

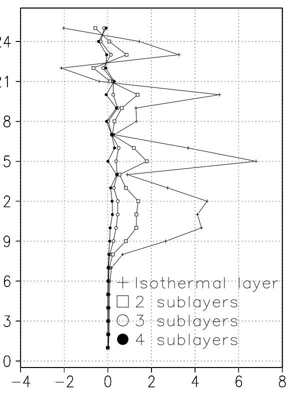


Figure 5: Error in Kelvin per day for exchanges between layer $i$ and $i+1$ computed with different numbers of sub-layers.

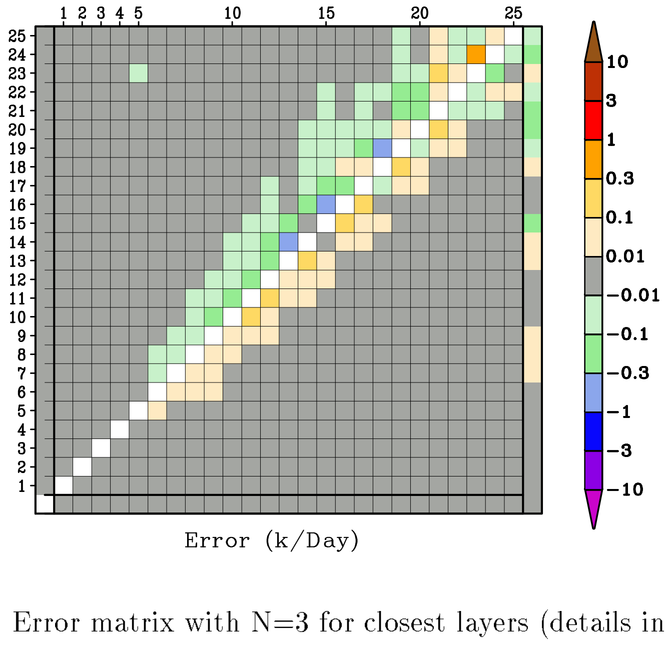


Figure 6: Error matrix with N=3 for closest layers (details in Fig. 4).

## 4 3D tests

The use of old or new code leads to temperature differences which are small with regard to other sources of uncertainties, such as dust radiative properties and spatial distribution.

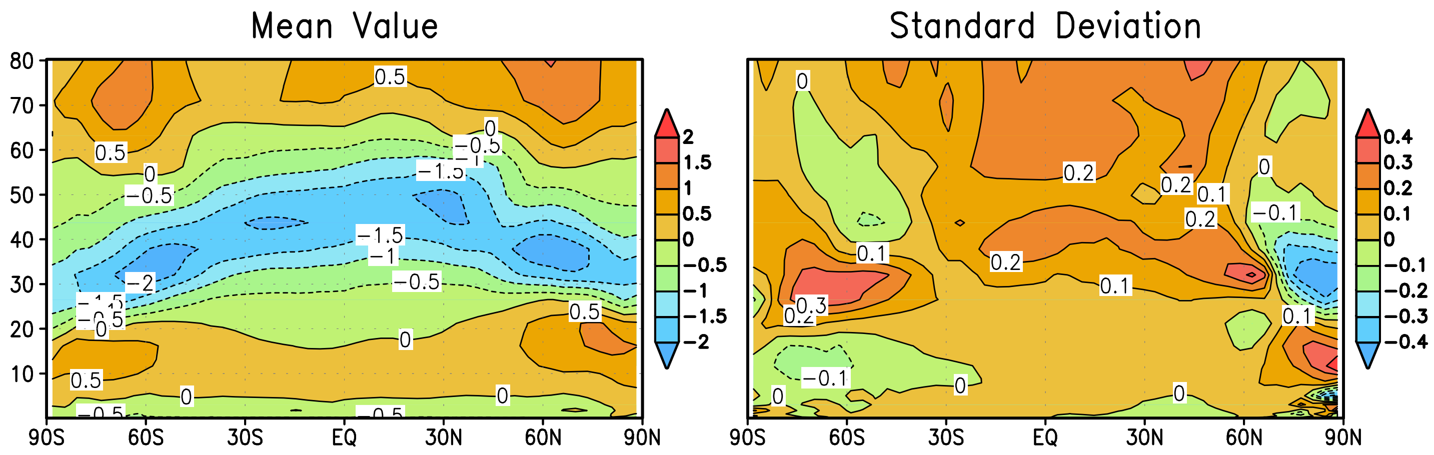


Figure 7: Temperature differences in Kelvin between old and new code for ten days simulations.

## 5 Saving computer time

The figure below displays error on mean zonally average temperature in Kelvin when exchange coefficiants are not computed each time step, with regard to simulation where exchange coefficiants are computed 48 times per day (the usual frequency, like in the original model). Each graph is the difference between results of 10 days simulations.

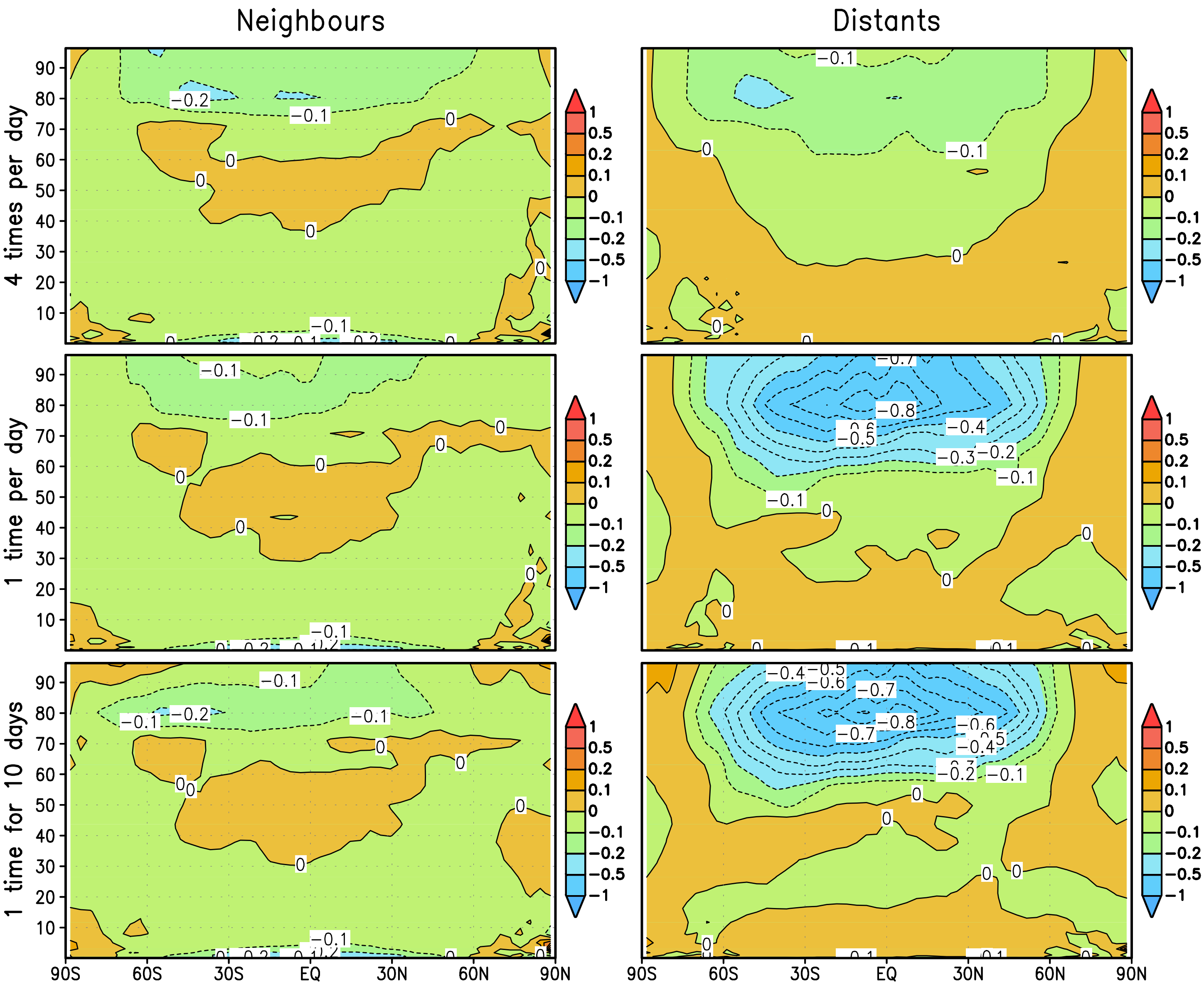


Figure 8:

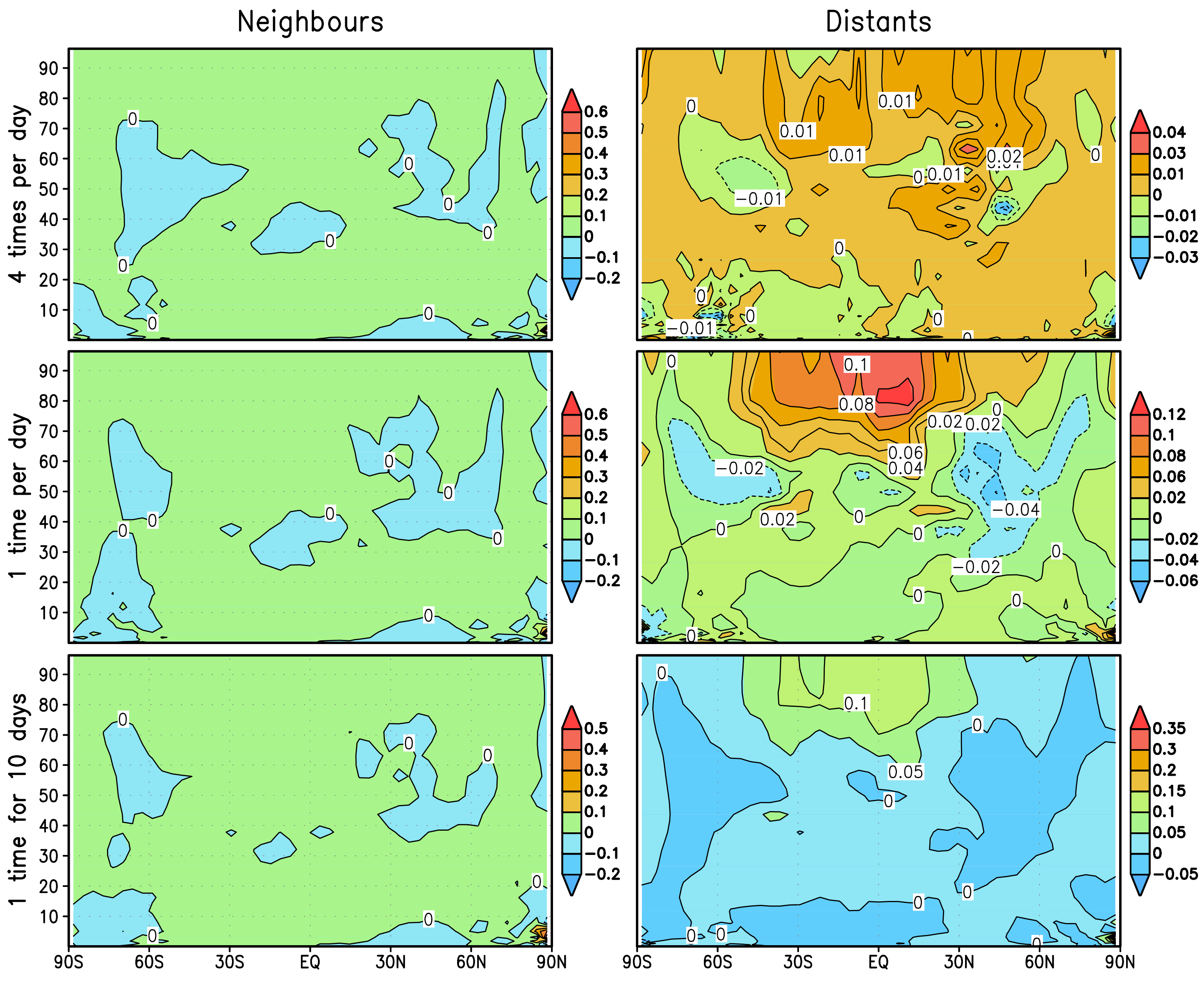


Figure 9: Same thing for Standard Deviation Values.

# 6 Conclusion

Thanks to our new approach, we have been able to separate computations between optical and thermal aspects. In our context we find that optical coefficients vary slowly in time. Since the optical part of the model is the most expensive numerical cost, it allows us to reduce the cost of the full radiative code by an order of magnitude without decreasing its accuracy. This will allow to increase the vertical resolution with a reasonable numerical cost.

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