

MODELING IONIZATION, ELECTRON-ION RECOMBINATION, AND VERTICAL DIFFUSION IN AN ISOTHERMAL MARTIAN IONOSPHERE

D. L. Huestis, *SRI International, Menlo Park, CA, USA (david.huestis@sri.com)*. [MP10-083]

Introduction:

We have recently shown [1] that the altitude profiles of $\text{CO}_2^+(\text{B}^2\Sigma_u^+ \rightarrow \text{X}^2\Pi_g)$ and $\text{CO}(\text{a}^3\Pi \rightarrow \text{X}^1\Sigma^+)$ Martian ultraviolet dayglow emissions above 130 km follow the altitude profile of ambient CO_2 , reflecting the temperature of the neutral atmosphere. In contrast, the $\text{O}(\text{S} \rightarrow \text{P})$ ultraviolet dayglow emissions above 130 km follow the altitude dependence of ion-electron recombination in the Mars ionosphere, reflecting the square of the electron density. Thus Mars dayglow and radio science ionospheric occultation measurements return strongly correlated information. The high altitude $\text{O}(\text{S})$ dayglow emissions teach us about the topside ionosphere plasma scale height and temperature, while the $\text{CO}_2^+(\text{B}^2\Sigma_u^+)$ and $\text{CO}(\text{a}^3\Pi)$ dayglow emissions teach us about the altitude profile of the primary ionizable atmospheric constituent, CO_2 .

Relatively few Mars missions have included spectrometers capable of recording ultraviolet dayglow emissions, while every Mars orbiter or near fly-by mission implicitly records information about the electron density altitude profile in the Mars ionosphere by refraction and time-delay just before radio transmission to Earth is blocked by the planet body and just after it becomes unblocked.

Here we report development of a parameterized analytical model of the time and altitude variation of the electron density in Mars ionosphere based on a new eigen-solution expansion. We outline analytical and numerical mathematical formulas, tools, and procedures for systematic automated analysis and fitting of electron density altitude profiles, versus Mars astronomical variables: solar zenith angle, solar activity, and heliocentric distance. The primary targeted dataset is the several thousands of electron density altitude profiles recorded by the *Mars Global Surveyor* (MGS) orbiter achieved in the Planetary Data System (PDS).

Eigen-Solutions of the Diffusion Equation:

Consider an isothermal one-dimensional atmosphere with a vertical diffusion coefficient given by $D(z) = D_0 \exp(z/H_D)$. We write the partial differential equation for diffusion of a trace minor chemical species with isothermal scale height H , as

$$(1) \frac{\partial}{\partial t} N(t,z) = \frac{\partial}{\partial z} \left\{ D_0 \exp(z/H_D) \left[\frac{\partial}{\partial z} N(t,z) + N(t,z)/H \right] \right\},$$

which has the well-know time-stationary or $t \rightarrow \infty$ solution of the form

$$(2) N_0(t,z) \approx \text{constant} \times \exp(-z/H).$$

We make an independent variable substitutions for time as $\tau = t \{D_0 / (4H_D^2)\}$. We also make the much more important substitution for altitude, $\xi = \exp\{-z / (2H_D)\}$, as suggested in the literature [2-5].

Using several definitions and equations in chapters 9-11 on Bessel functions in the handbook by Abramowitz and Stegun [6], we write the general solution of equation (1) as

$$(3) N(t,z) = \int_0^\infty g(\lambda) \exp(-\lambda^2 \tau) \xi^{\nu+2} J_\nu(\lambda \xi) d\lambda,$$

where $\nu = H_D/H - 1$ [5]. Ferraro and Özdoğan [2] were the first to find a Bessel function solution and L'vova *et al.* [5] were the first to implement Bessel function solutions with the order depending on the ratio of chemically unrelated scale heights. Bessel functions of integer and half-integer orders ν , are easily calculated from published formulas [6] and FORTRAN subroutines [7]. Subroutines for arbitrary orders are also available [8].

The non-zero values of $g(\lambda)$ in equation (3) can be determined from the fact that diffusion cannot create or destroy population. Thus $\lambda_0 = 0$ and $J_{\nu+1}(\lambda_j) = 0$, for $j > 0$, defining the eigen-solutions

$$(4) N_j(t,z) \approx \exp(-\lambda_j^2 \tau) \xi^{\nu+2} J_\nu(\lambda_j \xi),$$

the orthonormality relationships

$$(5) \int_0^\infty e^{z/H} N_j(0,z) N_k(0,z) dz = \delta_{jk},$$

and the complete solution of equation (1) for arbitrary initial conditions:

$$(6) N(t,z) = \sum_j c_j N_j(t,z), \text{ where}$$

$$(7) c_j = \int_0^\infty e^{z/H} N(0,z) N_j(0,z) dz.$$

Diffusion with Sources and Sinks:

Now we write

$$(8) N(t,z) = \sum_j c_j(t) N_j(0,z), \text{ where}$$

$$(9) c_j(0) = \int_0^\infty e^{z/H} N(0,z) N_j(0,z) dz.$$

Equation (1) then becomes

$$(10) \frac{\partial}{\partial t} N(t,z) = \frac{\partial}{\partial z} \left\{ D_0 \exp(z/H_D) \left[\frac{\partial}{\partial z} N(t,z) + N(t,z)/H \right] \right\} + \text{Source}(t,z) - \text{Sink}[z,N].$$

Substituting equation (8) into equation (10) we get

$$(11) \sum_j \frac{d}{dt} c_j(t) N_j(0,z) = - \left[D_0 / (4H_D^2) \right] \sum_j \lambda_j^2 c_j(t) N_j(0,z) + \text{Source}(t,z) - \text{Sink}[z,N].$$

Suppose that $\text{Sink}[z,N]$ has the general form

$$(12) \text{Sink}[z,N] = [\beta_0 + \beta_1(z)] N(t,z) + \beta_2 N(t,z)^2.$$

Then we multiply both sides of equation (11) $e^{z/H} N_j(0,z)$ and integrate using equation (5) to obtain

$$(13) \frac{d}{dt} c_i(t) = - [D_0 / (4H_D^2)] \lambda_i^2 c_i(t) + S_i(t) - \beta_0 c_i(t) - \sum_k c_k(t) R_{ik} - \beta_2 \sum_{jk} c_j(t) c_k(t) Q_{ijk}$$

where

$$(14) S_i(t) = \int_0^\infty e^{z/H} N_i(0,z) \text{Source}(t,z) dz ,$$

$$(15) R_{ik} = \int_0^\infty e^{z/H} N_i(0,z) \beta_1(z) N_k(0,z) dz , \text{ and}$$

$$(16) Q_{ijk} = \int_0^\infty e^{z/H} N_i(0,z) N_j(0,z) N_k(0,z) dz .$$

In the simplest case, where $\beta_{1,2}$ and β_2 are both zero, we write $\gamma_j = [D_0 / (4H_D^2)] \lambda_j^2 + \beta_0$, and find the following time-dependent solution [9]:

$$(17) c_i(t) = \exp(-\gamma_i t) [c_i(0) + \int_0^t S_i(t') \exp(\gamma_i t') dt']$$

Chamberlain [10, 11] and Chamberlain and Hunten [12] described an eigen-solution basis-function approach for the case of time decay of the terrestrial nighttime ionosphere, with no ionization source, i.e. $\text{Source}(t,z) = 0$, and terrestrial upper ionosphere electron-ion recombination rates that were inferred empirically [13-16] to be exponentially dependant on altitude but linear in the electron and ion densities, i.e. β_0 and β_2 are both zero, and $\beta_1(z) \approx \exp(-z/H_R)$, for the special case of $H_R = H_D$.

Glidden [16] had earlier investigated a similar case with a Chapman ionization source. Tuan [17] adopted a similar approach, along with comparisons to ionospheric observations. A number of authors [3, 4, 18-20] have added an ion-drift term of the form $-w \partial N / \partial z$. Makeev *et al.* [20] were the first to consider the case where all three scale heights, H , H_D , and H_R , are unrelated to each other.

In this case we rewrite equation (13) as

$$(18) \frac{d}{dt} c_i(t) = - [D_0 / (4H_D^2)] \lambda_i^2 c_i(t) + S_i(t) - \sum_k c_k(t) R_{ik}$$

This inhomogeneous linear system of first-order differential equations with constant coefficients can be solved explicitly by matrix manipulation and the *method of variation of parameters* [9] or treated as if a system of simple chemical interconversion reactions, for which efficient numerical integration packages are available [21].

Quadratic Electron-Ion Recombination in the Mars Ionosphere:

As indicated above, the focus of the work reported here is on modeling and fitting the altitude and time dependencies of the measured electron and ion densities in the Mars ionosphere versus solar activity, heliocentric distance, and local solar time [1]. The major positive ion is O_2^+ and electron-ion recombination is expected to be quadratic in the electron density, i.e., in equation (12), β_0 and β_1 vanish, but β_2 does not. Ferraro [22] made some early attempts to model the complicated quadratic recombination case. Equation (13) then becomes

$$(19) \frac{d}{dt} c_i(t) = - [D_0 / (4H_D^2)] \lambda_i^2 c_i(t) + S_i(t) - \beta_2 \sum_{jk} c_j(t) c_k(t) Q_{ijk}$$

Given initial conditions provided by equation (9), equation (19) can be solved by efficient chemical kinetics integration routines [21]. Alternatively, a steady state solution can be found by a nonlinear vector root search.

References:

- [1] D. L. Huestis, *et al.*, Faraday Disc. **147**, 307 (2010).
- [2] V.C.A. Ferraro and I. Özdoğan, J. Atmosph. Terr. Phys. **12**, 140 (1958).
- [3] J.E.C. Gliddon, Planet. Space Sci. **9**, 815 (1962).
- [4] J.E.C. Gliddon and P. C. Kendall, J. Atmosph. Terr. Phys. **24**, 1073 (1962).
- [5] A. A. L'vova, *et al.*, Izvestia Vysshikh Uchebnykh, Radiofizika, **15**, 840 (1972).
- [6] M. Abramowitz and I. A. Stegun, Eds., *Handbook of Mathematical Functions* (U.S. Gov. Print. Office, Washington, DC, 1970)
- [7] W. H. Press, *et al.*, *Numerical Recipes* (Cambridge Univ. Press, Cambridge, UK, 1986).
- [8] D. E. Amos, ACM Trans. Math. Software **12**, 265 (1986).
- [9] A. L. Rabenstein, *Introduction to Differential Equations* (Academic Press, New York, 1966),
- [10] J. W. Chamberlain, *Physics of the Airglow and Aurora* (Academic Press, New York, 1961), & Reprint Edition (Am. Geophys. Union, Washington, DC, 1995).
- [11] J. W. Chamberlain *Theory of Planetary Atmospheres*, An Introduction to Their Physics and Chemistry, 1st Ed. (Academic Press, New York, 1978).
- [12] J. W. Chamberlain and D. M. Hunten, *Theory of Planetary Atmospheres*, 2nd Ed. (Academic Press, San Diego, 1987).
- [13] N. E. Bradbury, Terr. Magn., **43**, 55 (1938).
- [14] D. R. Bates and H.S.W. Massey, Proc. Roy. Soc. (Lond.) **A187**, 261 (1946); **A192**, 1 (1947).
- [15] J. A. Ratcliffe, *et al.*, Phil. Trans. Roy Soc. (Lond.) **A248**, 621 (1956).
- [16] J.E.C. Gliddon, Quart. J. Mech. Appl. Math. **12**, 340 (1959); 347 (1959).
- [17] T. F. Tuan, J. Phys. B **1**, 966 (1968).
- [18] J. W. Dungey, J. Atmosph. Terr. Phys. **9**, 90 (1956).
- [19] D. F. Martyn, Proc. IRE **47**, 147 (1959).
- [20] V. V. Makeev, *et al.*, Izvestia Vysshikh Uchebnykh, Radiofizika, **16**, 1284 (1973).
- [21] A. C. Hindmarsh, IEEE Control Systems Magazine, **2**, 24 (1982).
- [22] V.C.A. Ferraro, Terr. Magn. Atmosph. Electr. **50**, 215 (1945); **51**, 427 (1946).